ETH zürich

Random matchings in study-design: A graph-theoretical approach

Tassilo Schwarz
Center for Law and Economics, ETH Zurich

Agenda

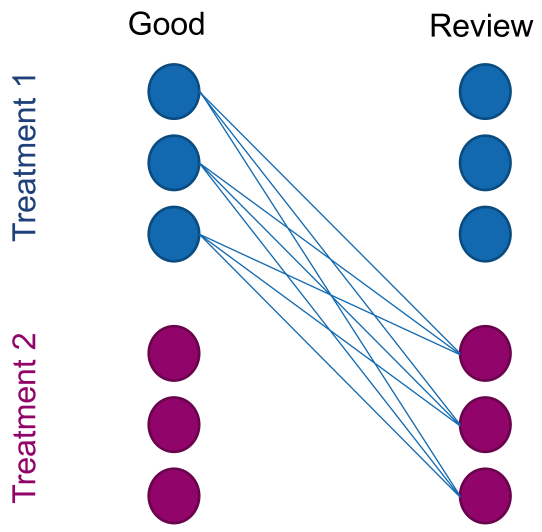
1. Motivating example
2. Translating to graph theory
3. The Issue: Why simple approaches fail
4. A very special case: k-partite, k-regular graphs
5. Outlook & Discussion

Motivating example: A two-phase experiment

- Phase 1:
  - Participants get partitioned into k treatment groups
  - Each participant produces some good
  - (For example, condition under which participants produce good could vary between treatments)
- Phase 2:
  - Goods get peer-reviewed by other participants => need to find a matching
  - Want to randomize: Every good-reviewer combination should be equally likely
  - (There may be some potential confounding factors, e.g. between 1st participant in group A and 1st participant in group B)

- Possible additional constraints:
  - No good is reviewed by participants from the same treatment group
  - Multiple reviewers per good
  - Multiple reviewers should be equally spread among treatment groups

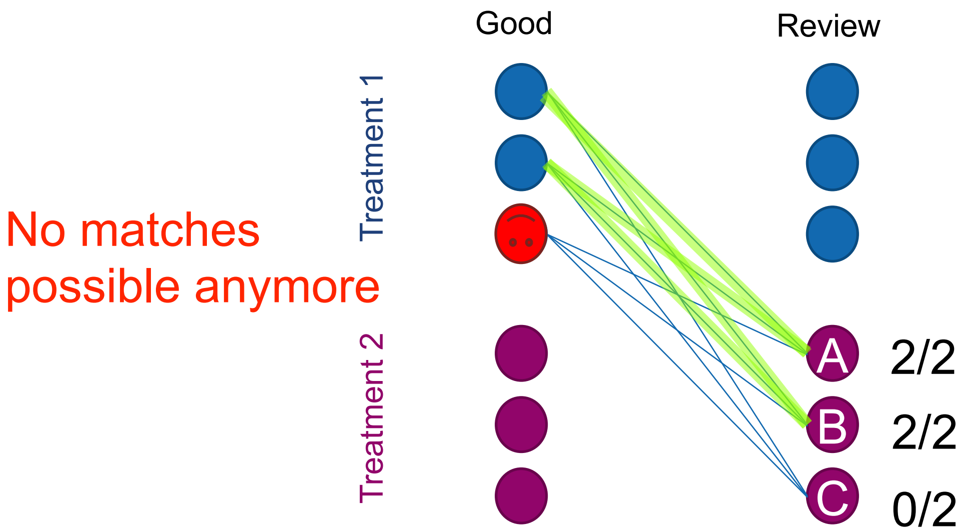
Translating to graph theory



Constraints:

- No good is reviewed by participants from the same treatment group
- Two different reviewers per good, each reviewer reviews two different goods

The issue: Why simple approaches fail



=> Use flow graph in deterministic case

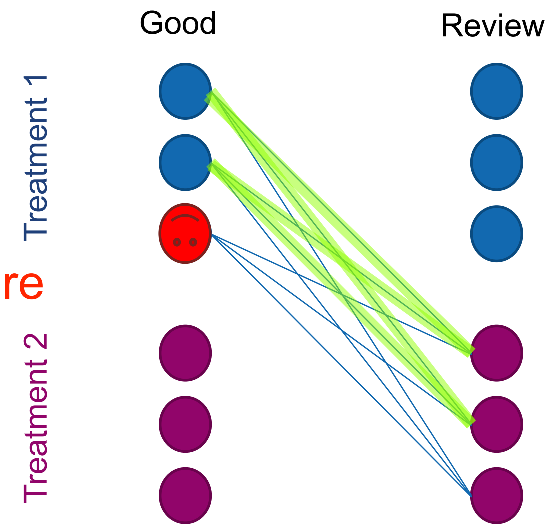
Constraints:

- No good is reviewed by participants from the same treatment group
- Two different reviewers per good, each reviewer reviews two different goods

Algorithm, design attempt:

- 1: Select Good:
  - Select group g at random
  - Select good x of g at random
- 2: Select Reviewer:
  - Select group h at random (h ≠ g)
  - Select reviewer y of h at random
  - If reviewer has maximum degree, goto 2
- Match (x,y)
- Recurse until eveyone is matched

The issue: Why simple approaches fail



No matches possible anymore

Results from complexity theory

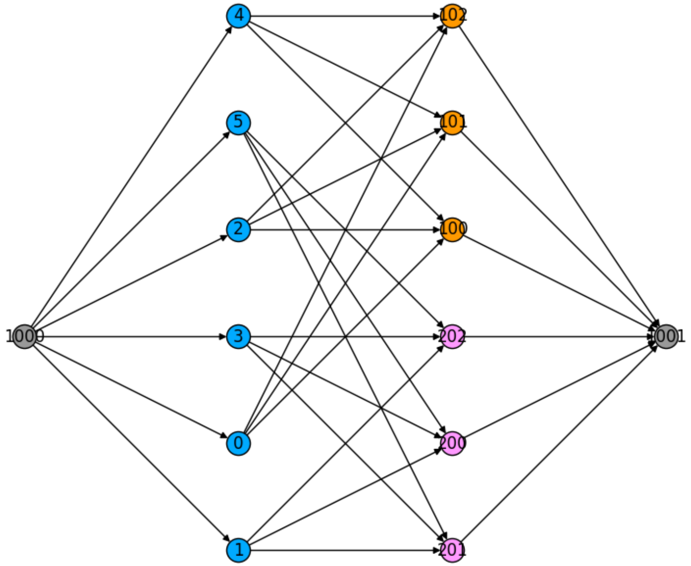
– Intuitively: Sampling uniformly connected to counting (sampling with p = $\frac{1}{M}$ => sufficiently many samples gives ≈ M)
– Exact counting in bipartite case: #P-complete (Valiant 1979). Solved? Call Clay institute to get 1.000.000$

Good news:

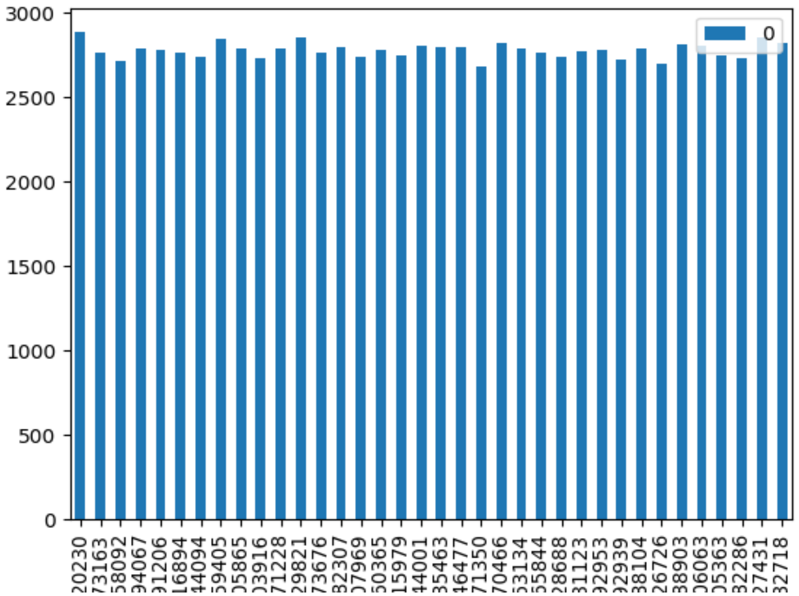
– Proven: Almost uniform sampling and approximately counting interreducible (Jerrum, Valiant and Vazirani 1986)
– Good approximations for counting in bipartite graphs (Jerrum, Sinclair and Vigoda 2004)

The very special case: Solution for k-partite, k-regular

- Classical fix: Flow graphs.
- But: Not randomized
- For highly structured cases, can introduce randomness by shuffeling nodes



Flow graph for capacitated-b-matching in k partite, k regular case



Testing n=100.000 times. Each of the 36 possible perfect matchings is equally likeley

Solved,
for this case

Outlook & Discussion

- More general cases?
- Studies?
- Possible further applications?
  - Entry-level labour markets (Roth et al. 1990)
  - Problem: Not uniformly random. Some matches occur even with probability p=0 (Ma 1996)

References

- Leslie G Valiant. The complexity of computing the permanent. Theoretical computer science, 8(2):189–201, 1979.

- Ma 1996: Jinpeng Ma. On randomized matching mechanisms. Economic Theory, 8(2):377–381, 1996.
- Mark Jerrum, Alistair Sinclair, and Eric Vigoda. A polynomial-time approximation algorithm for the permanent of a matrix with nonnegative entries. J. ACM, 51(4):671–697, July 2004.
- Mark R Jerrum, Leslie G Valiant, and Vijay V Vazirani. Random generation of combinatorial structures from a uniform distribution. Theoretical computer science, 43:169–188, 1986.
- Roth et al. 1990: Alvin E Roth and John H Vande Vate. Random paths to stability in two-sided matching. Econometrica: Journal of the Econometric Society, pages 1475–1480,1990.

- Extensive bibliography for matchings and market design: https://stanford.edu/~alroth/matchbib.html